Photo-assisted shot noise in the fractional quantum Hall regime

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Abstract.
The effect of an ac perturbation on the shot noise of a fractional quantum Hall fluid is studied at finite temperature. For a normal metal, it is known that the zero-frequency noise derivative exhibits steps as a function of bias voltage. In contrast, at Laughlin fractions, the backscattering noise exhibits evenly spaced singularities, which are reminiscent of tunneling density-of-states singularities for quasiparticles. The spacing is determined by the quasiparticle charge νe and the ratio of the dc bias with respect to the drive frequency. Photo-assisted transport can thus be considered as a probe for effective charges of the quantum Hall effect.

Keywords: Shot noise, Luttinger liquid, Edge states, Photo-assisted transport.
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INTRODUCTION

In mesoscopic systems, the measurement of shot noise makes it possible to probe the effective charges which flow in conductors, and opens the possibility for studying the role of the statistics in stationary quantum transport experiments. This has been illustrated experimentally and theoretically where the interaction between electrons is less important[1, 2, 3, 4, 5, 6] or when it is more relevant[7, 8, 9, 10, 11]. The present work deals with the study of photo-assisted shot noise in a specific one dimensional correlated system: a Hall bar in the fractional quantum Hall regime, for which charge transport occurs via two counter-propagating chiral edges states.

MODEL

We consider the system depicted on Fig. 1 which is described by the Hamiltonian:

$$H = \frac{\hbar v_F}{4\pi} \sum_{r=R,L} \int ds (\partial_s \phi_r(t))^2 + A(t)\Psi_R^\dagger(t)\Psi_L(t) + A^*(t)\Psi_L^\dagger(t)\Psi_R(t)\ . \tag{1}$$

The bosonic fields $\phi_{R(L)}$, which describe the right and left moving chiral excitations along the edge states, are related to the fermionic fields $\Psi_{R(L)}$ through:

$$\Psi_{R(L)}(t) \quad = \quad \frac{F_{R(L)}}{\sqrt{2\pi a}} e^{i\sqrt{\nu}\phi_{R(L)}(t)}\ , \tag{2}$$

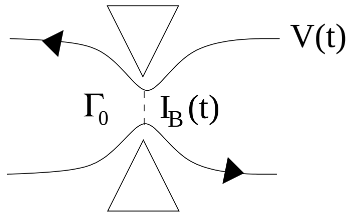

FIGURE 1. Backscattering between edge states in the presence of a bias voltage modulation $V(t)$.

where $F_{R(L)}$ is a Klein factor, a, the short-distance cutoff and ν, the filling factor which characterize the charge $e^* = \nu e$ of the backscattered quasiparticles. The hopping amplitude between the edge states has a time-dependence due to the applied voltage $V(t) = V_0 + V_1 \cos(\omega t)$:

$$A(t) = \Gamma_0 \sum_{n=-\infty}^{+\infty} J_n\left(\frac{\omega_1}{\omega}\right) e^{i(\omega_0 + n\omega)t} \;, \tag{3}$$

where we have made an expansion in term of an infinite sum of Bessel functions, which is a signature of photo-assisted processes[12]. It is important to notice that the frequencies ω_0 and ω_1 which appear in Eq. (3) are related to the filling factor ν:

$$\omega_0 \equiv \nu e V_0/\hbar \;, \qquad \omega_1 \equiv \nu e V_1/\hbar \;. \tag{4}$$

where $\nu = 1/(2m+1)$ with m integer.

PHOTO-ASSISTED SHOT NOISE

The symmetrized backscattering current noise correlator is expressed with the help of the Keldysh contour:

$$\begin{aligned} S(t,t') &= \frac{1}{2}\langle I_B(t) I_B(t')\rangle + \frac{1}{2}\langle I_B(t') I_B(t)\rangle - \langle I_B(t)\rangle\langle I_B(t')\rangle \\ &= \frac{1}{2}\sum_{\eta=\pm 1} \langle T_K\{I_B(t^\eta) I_B(t'^{-\eta}) e^{-i\int_K dt_1 H_B(t_1)}\}\rangle \;, \end{aligned} \tag{5}$$

where H_B is the sum of the second and the third terms in Eq. (1), and:

$$I_B(t) = \frac{i\nu e}{\hbar} A(t) \Psi_R^\dagger(t) \Psi_L(t) - h.c. \tag{6}$$

We are interested in the Poissonian limit only, so in the weak backscattering case, one collects the second order contribution in the tunnel barrier amplitude $A(t)$, and the product of the average backscattering currents can be dropped. The meaning of the Poissonian limit is that quasiparticles which tunnel from one edge to another do so in an independent manner. Yet by doing so they can absorb or emit n “photon” quanta of

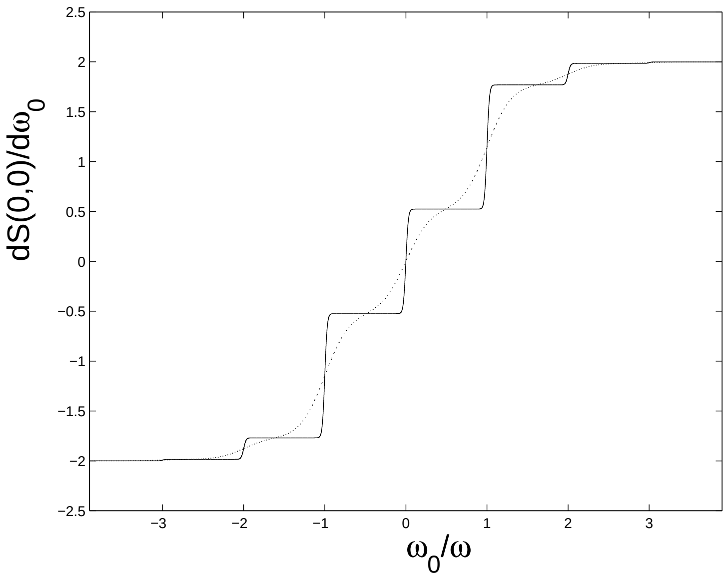

FIGURE 2. Noise derivative for a normal metal at different temperatures: $k_B T/\hbar\omega = 0.01$ (solid line) and $k_B T/\hbar\omega = 0.1$ (dashed line). We take $\omega_1/\omega = eV_1/\hbar\omega = 3/2$.

ω (n integer). The main purpose of this work is to analyze the double Fourier transform of the noise $S(\Omega_1,\Omega_2) \propto \int dt \int dt' S(t,t') \exp(i\Omega_1 t + i\Omega_2 t')$ when both frequencies Ω_1 and Ω_2 are set to zero. Indeed, the presence of the AC perturbation mimics a finite frequency noise measurement. At zero temperature, the shot noise exhibit divergences at each integer value of the ratio ω_0/ω[13]. These divergences are not physical since they appear in a range of frequencies where the perturbative calculation turn out to be no more valid. For this reason, we have performed finite temperature calculations which prevent divergences in the backscattering current and shot noise. At finite temperature, the shot noise reads:

$$\begin{aligned} S(0,0) &= \frac{(e^*)^2\Gamma_0^2}{2\pi^2 a^2 \overline{\Gamma}(2\nu)} \left(\frac{a}{\mathrm{v}_F}\right)^{2\nu} \left(\frac{2\pi}{\beta}\right)^{2\nu-1} \\ &\quad \times \sum_{n=-\infty}^{+\infty} J_n^2\left(\frac{\omega_1}{\omega}\right) \cosh\left(\frac{(\omega_0+n\omega)\beta}{2}\right) \left|\overline{\Gamma}\left(\nu + i\frac{(\omega_0+n\omega)\beta}{2\pi}\right)\right|^2 , \quad (7) \end{aligned}$$

where $\overline{\Gamma}$ is the Gamma function and $\beta = 1/k_B T$.

DISCUSSION

We have first test the validity of our result by setting $\nu = 1$, the value which corresponds to non-interacting system (normal metal). The derivative of the shot noise according to the bias voltage exhibits staircase behavior as shown on Fig. 2. Steps occur every time ω_0 is an integer multiple of the ac frequency. This is in complete agreement with the

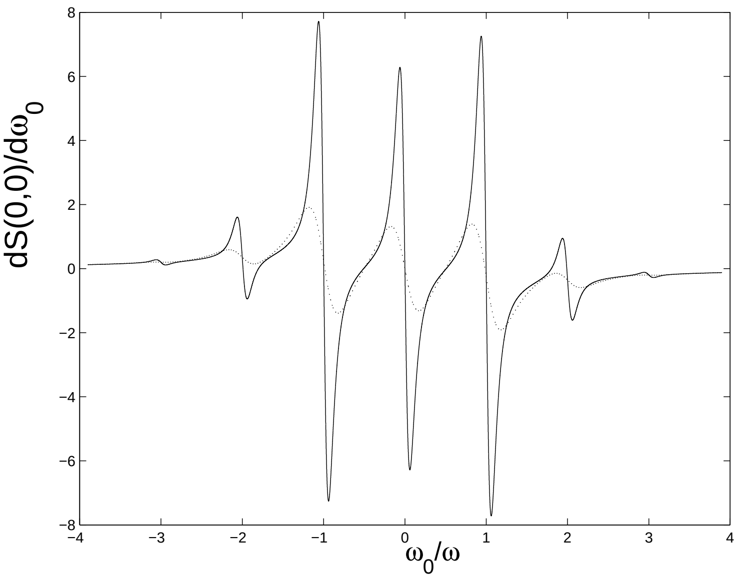

FIGURE 3. Noise derivative in the fractional quantum Hall regime at a filling factor $\nu = 1/3$ for $k_BT/\hbar\omega = 0.05$ (solid line) and $k_BT/\hbar\omega = 0.15$ (dotted line). We take $\omega_1/\omega = \nu eV_1/\hbar\omega = 3/2$.

results obtained be Lesovik and Levitov for a Fermi liquid[14]. When the temperature increases, the steps are rounded.

For non-integer value of the filling factor ($\nu = 1/3$, for example), the shot noise derivative exhibits evenly spaced singularities, which are reminiscent of the tunneling density of states singularities for Laughlin quasiparticles. The spacing is determined by the quasiparticle charge νe and the ratio of the bias voltage with respect to the ac frequency, and the amplitude is governed by temperature (see Fig. 3). Photo-assisted transport can thus be considered as a probe for effective charges at such filling factors, and could be used in the study of more complicated fractions of the quantum Hall effect.

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